Calculators and mobile phones are not allowed.

Answer all of the following questions.

Each (sub)question is worth 5 points.

1. Find the limit

$$\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}.$$

2. Evaluate the integrals

(a) $\displaystyle\int \tan^3 x \cot x \sin x \, dx,$

(b) $\displaystyle\int \frac{dx}{x^4\sqrt{x^2 - 9}},$

(c) $\displaystyle\int \frac{4 \, dx}{1 + x - x^2 - x^3},$

(d) $\displaystyle\int 3x^5 \sin x^3 \, dx.$

3. Determine if the following improper integral converges or diverges, and if it converges find its value

$$\int_0^\infty \frac{x^2}{(1 + x^3)^2} dx.$$

4. Find the arc length of the parametric curve

$$x = \cos^3 t$$
$$y = \sin^3 t \qquad (0 \le t \le 2\pi).$$

5. Sketch the polar curve $r = 2 - 2\cos\theta$, $\theta \in [0, 2\pi]$, label at least 4 points on its graph and find the slope of the tangent line at the point $P(\frac{\pi}{3}, 1)$.